**Quiz #3:** 4.1-4.8, 5.1, 5.2, 5.6-5.8

**4.2 #15**

Prove the product of any two rational numbers is a rational number or provide a counterexample.

A = v/w

B = x/y

w, y > 0

v,w,x,y are integers

v\*x/w\*y

Since a and b are rational a\*b is rational. True.

**4.4 #19 (HW4)**

Proof:

Suppose n is any integer. By the quotient remainder theorem with d = 2, n = 2q is even or n = 2q + 1 is odd.

*Case 1 (n is even):* n = 2q for some integer q.

n² - n + 3

= (2q) ² - 2q + 3 by substitution, let n = 2q

= 4q² - 2q + 2 + 1

= 2(2q² - q + 1) + 1 by algebra

Let x = (2q² - q + 1). Then x is an integer since products, sums and differences of integers are integers. Thus, substituting,

n² - n + 3 = 2x + 1 definition of odd

*Case 2 (n is odd):* n = 2q + 1 for some integer q.

n² - n + 3

= (2q + 1) ² - 2q + 1 + 3 by substitution, let n = 2q + 1

= 4q² + 4q - 2q + 2 + 3

= 4q² + 2q + 4 + 1

= 2(2q² + q + 2) + 1 by algebra

Let x = (2q² + q + 2). Then x is an integer since products and sums of integers are integers. Thus, substituting,

n² - n + 3 = 2x + 1 definition of odd

Thus for all integers n, n² - n + 3 is an odd integer.

□

**4.6 #21**

Consider the statement “For all integers n, if n2 is odd then n is odd”

1. Write what you would suppose and what you would need to show to prove this statement by contradiction
2. Write what you would suppose and what you would need to show to prove this statement by contraposition.
3. N2 is odd, n is even

N = 2k

N2 = 2k \* 2k =/ 2(2k2) + 1

1. N is not odd or n is even, n2 not odd

N = 2k

N2 = 2(2k2)

N2 is even

**5.2 #10**

Prove with induction:

12 + 22 +···+ n2 = [n(n + 1)(2n + 1)]/6 , for all integers n ≥ 1.

P(1), for n = 1 Substituting P(1) = 1

12 + …. + k2 + (k+1)2 = [k(k+1)(2k +1)]/6 + (k+1)2

= [k(k+1)(2k +1)6(k+1)2]/6

= [(k+1)(k+1+1)(2(k+1)+1)]/6

Substituting k+1 = n we have original RHS equation

**5.6 #20a (HW5) HW Solutions**

**1.** Move K-1 disks from A to B, the biggest disk cannot be moved yet.

Ck-1 steps to move K-1 disks

**2.** Repeat step 1 for B to C

Total moves = Ck-1 + Ck-1

**3.** Move biggest disk from A to B

Total moves = Ck-1 + Ck-1 + 1

**4.** Repeat step 1 for C to A

Total moves = Ck-1 + Ck-1 + 1 + Ck-1

**5.** Repeat step 1 for A to B

Total moves = Ck-1 + Ck-1 + 1 + Ck-1 + Ck-1

Max Ck = 4Ck-1 + 1 (Max number of moves)

Therefore, Ck ≤ 4 ∙ Ck-1 + 1

**WRONG**

Using k = 2 and since C1 = 1 and C2 = 5, the minimum number of steps required to move two disks,

C2 ≤ 4 ∙ C1 + 1

5 ≤ 4 ∙ 1 + 1

5 ≤ 5

With C3 = 15, the minimum number of steps required to move three disks,

C3 ≤4 ∙ C2 + 1

15 ≤ 4 ∙ 5 + 1

15 ≤ 21

So, for all integers k ≥ 2, the inequality Ck ≤ 4 ∙ Ck-1 + 1 holds.

**5.7 #13 (HW5) Simplifications in 5.2, HW5 Solutions**

Tk = Tk-1 + 3k + 1, for all k ≥ 1 and T0 = 0

Tk-1 = Tk-2 + 3(k – 1) + 1 (equation 13.1)

Tk-2 = Tk-3 + 3(k – 2) + 1 (equation 13.2)

Tk-3 = Tk-4 + 3(k – 3) + 1 (equation 13.3)

Tk = Tk-1 + 3k + 1

= Tk-2 + 3(k – 1) + 1 + 3k + 1 (substituting 13.1 for Tk-1)

= Tk-2 + 3[(k – 1) + k] + 1 ∙ 2

= Tk-3 + 3(k – 2) + 1 + 3[(k – 1) + k] + 1 ∙ 2 (substituting 13.2 for Tk-2)

= Tk-3 + 3[(k – 2)(k – 1) + k] + 1 ∙ 3

Tk-k + 3[(k(k+1))/2] + 1 ∙ k

= 0 + 3[(k(k+1))/2] + 1 ∙ k (since Tk-k = T0 = 0)

= 3[(k(k+1))/2] + k

**5.8 #12 (HW5)**

This is a distinct roots case, using Theorem 5.8.3, t2 – 0t – 9 = 0 is the characteristic equation. Factoring leads to (x – 3)(x + 3) = 0 so, r = 3 and s = -3.

En = C ∙ 3n + D ∙ (-3)n

E0 = 0 = C + D so, C = -D

E1 = 2 = C ∙ 31 + D ∙ (-3)1

2 = -3D – 3D (substituting C = -D)

D = -1/3

Therefore, C = 1/3 and the formula is Ek = (1/3) ∙ 3k + (-1/3) ∙ (-3)k for all integers k ≥ 2

**Can simplify more:** -3(k-1) + 3(k-1)